

# Edge Extrusion Approach to Generate the Extruded Miura-Ori and Double Tiling Patterns

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## Abstract

Origami core sandwich panels are used as lightweight high-stiffness material [1]. The core material can be manufactured by folding a piece of planar material. However, when it is bonded on skin-layers, bonding regions are merely edges with zero areas, potentially making the bonding between the core and the skin weak [2] (Figure1 (a)). This paper presents an approach to generate the face-bonding sandwich core (Figure1 (b)). We call it extruded Miura-ori because it is obtained by cutting Miura-ori apart along a path that consist of edges and diagonal lines of parallelograms and then inserting the extrusion of the section edges (Figure2 (a)). We can control its shape by adjusting parameters (1; extrusion depth, 2; folding angle of Miura-ori, 3, 4; shape of Miura-ori parallelogram whose parameters are represented by sector angle and length's ratio of width to oblique side) which are independent up to scaling. Note that the extrusion direction is decided to satisfy the developable condition. When we define the notations as below (and see Figure2 (b)),

$\mathbf{n}$ : Unit extrusion vector,

$\mathbf{e}_1, \mathbf{e}_2$ : Unit section-edge vectors,

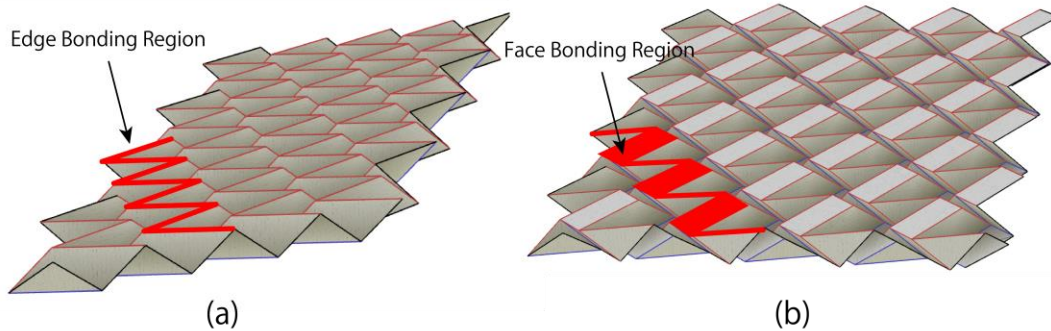
$\theta_1, \theta_2$ : Sector angles between  $\mathbf{n}$  and  $\mathbf{e}_1, \mathbf{e}_2$ ,

$\theta_{inner}$ : Sum of inner angles of an extruded surface,

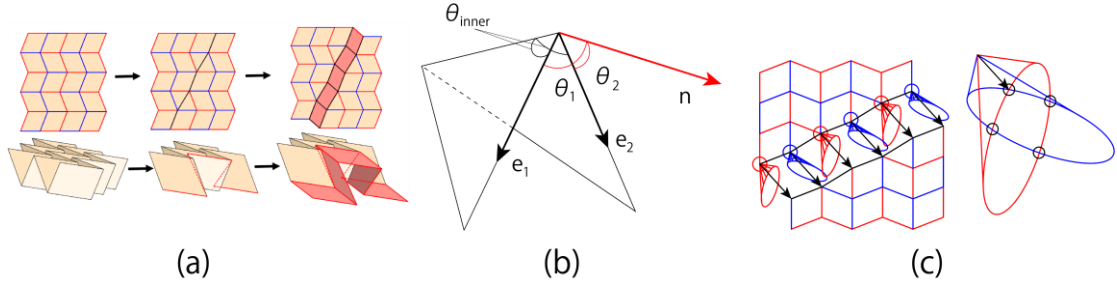
the developable condition can be expressed as

$$((\cos \theta_{inner})^2 - 1)||\mathbf{n}||^2 + (\mathbf{e}_1 \cdot \mathbf{n})^2 + (\mathbf{e}_2 \cdot \mathbf{n})^2 - 2(\cos \theta_{inner})(\mathbf{e}_1 \cdot \mathbf{n})(\mathbf{e}_2 \cdot \mathbf{n}) = 0.$$

By this condition, the extrusion direction lies on elliptic cone. Thus, the direction is determined by the intersection of the cones if there are only two types of vertices (Figure2 (c)).

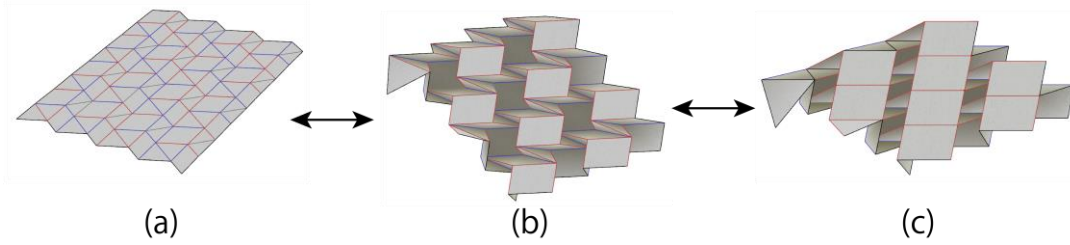


**Figure 1:** (a) Miura-ori. (b) Extruded Miura-ori.

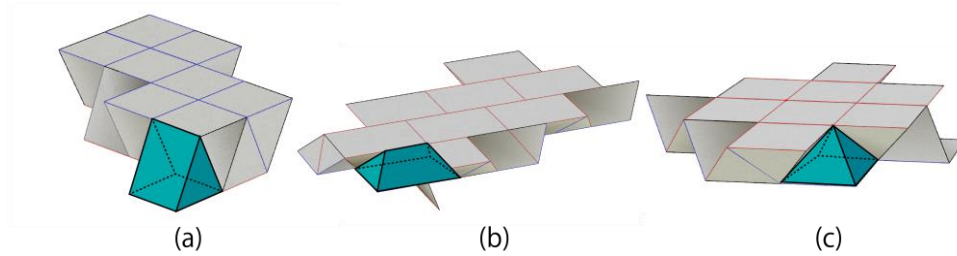


**Figure 2:** (a) Geometric construction of extruded Miura-ori. (b) Notations of developable condition. (c) Extrusion direction lies on the elliptic cone.

There are three folded states of extruded Miura-ori that make the top parallelograms lie on a plane and the bottom ones also on a plane. These folded states are (a) the developed state, (b) the state of geometric construction, and (c) the state where adjacent top parallelograms touch (Figure 3). We call (c) the final state as the contact between parallelograms blocks further folding. Under some conditions of parameters, the crease pattern can have a special final folded state in which the coplanar extruded parallelograms tile the two planes. The tiling pattern can be classified into three types. Namely prism type (Figure4 (a)), hip-roof type (Figure4 (b)), and pyramid type (Figure4 (c)).



**Figure 3:** Three flat-head folded states. (a) Developed state. (b) Mid state (extruded miura-ori state). (c) Final folded state.



**Figure 4:** (a) Prism type. (b) Hip-roof type. (c) Pyramid type.

## References

- [1] Miura K., “Zeta-Core Sandwich—Its Concept and Realization”. Institute of Space and Aeronautical Science Report, 1972; 137-164
- [2] Klett Y. and Drechsler K., “Designing Technical Tessellations”. Origami5, 2010, 305-321